Teachers' views about multiple strategies in middle and high school mathematics: Perceived advantages, disadvantages, and reported instructional practices

Running head: TEACHING WITH MULTIPLE STRATEGIES

Teachers' Views about Multiple Strategies in Middle and High School Mathematics

Kathleen Lynch and Jon R. Star

Harvard University

Abstract

Despite extensive scholarship about the importance of teaching mathematics with multiple strategies in the elementary grades, there has been relatively little discussion of this practice in the middle and high school levels or in the context of introductory algebra. This paper begins our exploration of this practice by addressing the following questions: (1) What do middle and high school Algebra I teachers describe as the advantages of instruction that includes a focus on multiple strategies?; and (2) What disadvantages to this practice do these teachers describe?. Our analysis, based on the data from interviews (N=13) and surveys (N=79) conducted with experienced middle and secondary mathematics teachers, indicates that middle and secondary math teachers' reported views surrounding multiple strategies appear to differ in important ways from those typically associated with teaching with multiple strategies in the elementary grades.

KEYWORDS: Multiple strategies, Algebra, Middle school, Secondary school

Teachers' Views about Multiple Strategies in Middle and High School Mathematics

The practice of teaching children multiple strategies for solving mathematics problems has been recommended in recent mathematics education policy reports in many countries (e.g., Australian Education Ministers, 2006; Brophy, 2000; Kultusministerkonferenz, 2004; National Council of Teachers of Mathematics, 2000; Singapore Ministry of Education, 2006; Treffers, 1991; Woodward et al., 2012). By multiple strategies, we refer to students' ability to solve mathematics problems in more than one way; as noted by Silver and colleagues, "It is nearly axiomatic among those interested in mathematical problem solving as a key aspect of school mathematics that students should have experiences in which they solve problems in more than one way" (Silver, Ghousseini, Gosen, Charalambous, & Strawhun, 2005, p. 288).

Although there has been extensive scholarship about the importance of and implementation of this practice in the elementary grades, there has been relatively little discussion of what teaching with multiple strategies would or should look like at the middle and high school levels. Should middle and high school mathematics teachers provide instruction emphasizing multiple strategies, or are the benefits of this instructional approach limited to the elementary grades? Are there differences in the ways that this practice can and should be implemented in middle and high school classrooms, as compared to elementary classrooms? Furthermore, despite the widespread identification of introductory algebra as a critical "gatekeeper" course for students' future education and employment prospects (e.g., Cai & Knuth, 2010; Izsák, Çağlayan, & Olive, 2009; Knuth, Alibabi, McNeil, Weinberg, & Stephens, 2010; Moses & Cobb, 2001), there has been little discussion of what multiple strategies instruction might or should look like in beginning algebra classrooms.

This paper begins our exploration of the practice of teaching students multiple strategies for solving mathematics problems at the middle and high school levels, in the context of a professional development institute whose goal was to encourage middle and high school Algebra I teachers to think more deeply and carefully about the use of multiple strategies in their instruction. The paper addresses the following questions: (1) What do middle and high school Algebra I teachers describe as the advantages of instruction that includes a focus on multiple strategies?; and (2) What disadvantages to this practice do these teachers describe? We examine these questions through interview and survey data collected from experienced middle and high school Algebra I teachers.

Our interest in teachers' views on teaching with multiple strategies is grounded in the literature on teachers' beliefs, particularly the relationship between what teachers believe about mathematics and mathematics teaching and their instructional practices. Philipp (2007) defined beliefs as "psychologically held understandings, premises, or propositions about the world that are thought to be true," but that, distinct from knowledge, "may be held with varying degrees of conviction" (p. 259). Researchers have further conceptualized beliefs as structured in complex systems, with individual beliefs intertwined with other beliefs (rather than held in isolation); more or less strongly held and subject to change; and held in what may be relatively isolated clusters, allowing individuals to hold what appear to be contradictory beliefs within the same belief system (Philipp, 2007; Thompson, 1992). Researchers must infer individuals' beliefs from their words, intentions, and actions (Pajares, 1992).

The research literature has additionally identified several elements of teachers' beliefs which are thought to play a key role in the way in which teachers' beliefs are related to their instructional practices. For example, Philipp (2007) noted that beliefs may be conceptualized as "lenses that affect one's view of some aspect of the world or as dispositions toward action" (p. 259). In this view, teachers' decisions about which actions to take in response to contextualized situations may provide one window (albeit a partial one) allowing inferences into their beliefs (Ambrose, Clement, Philipp, & Chauvot, 2004; Philipp, Ambrose, Lamb, Sowder, Schappelle, Sowder, Thanheiser, & Chauvot, 2007).

While the relationship between teachers' beliefs and their observed instructional practices is complex, with different aspects of teachers' beliefs made visible or enacted depending on elements of the interaction of the teacher with the teaching context (Thompson, 1992), research evidence suggests that teachers' beliefs regarding instructional practices play an important role in shaping their implementation of such practices in their classrooms (see reviews by Clark & Peterson, 1986; Fang, 1996; Kagan, 1992; Thompson, 1992). For example, studies conducted in multiple contexts suggest that teachers' beliefs relating to novel instructional practices play a role in determining the manner and extent to which they integrate the new practices into their instruction (e.g. Hollingsworth, 1989; Peterson, Fennema, Carpenter, & Loef, 1989; Richardson, 1990; Stylianou, 2010; Thompson, 1984; Wilkins, 2008). Since teachers' beliefs about reform practices in particular appear influential for their implementation of reform-oriented curricula, it is important to understand what beliefs teachers hold surrounding multiple strategies instruction, in order to discern what role these beliefs might play in supporting and/or preventing teachers from implementing instruction with multiple strategies effectively.

In the current study, we report on teachers' views about teaching with multiple strategies as expressed in their responses to interviews and surveys. While we developed interviews and survey measures as an inquiry into teachers' beliefs, we recognize that our interviews and surveys provide only a limited and partial picture of teachers' true underlying beliefs, which, as discussed above, are likely not visible and situated in highly complex and dynamic underlying core belief systems and structures (Thompson, 1992). As such, we use the term ‘views’ in the current study to describe the messages that teachers articulated in their interview and survey responses about their opinions, ideas, and beliefs regarding teaching and learning with multiple strategies, with the awareness that teachers’ responses can provide only a limited and exploratory picture of teachers’ underlying beliefs about multiple strategies learning and teaching.

Theoretical Framework

In considering teachers’ views about multiple strategies, one theoretical lens through which we can conceptualize these views is in terms of *responses to a policy reform*. As we discuss below, most teachers in the current study generally described teaching with multiple strategies as somewhat novel to them, in the sense that most stated they were not frequently exposed to multiple strategies for solving math problems in their own learning. At the same time, the teachers in the current study may be seen as having expressed some level of endorsement of the policy reform idea of exposing students to multiple strategies in instruction, given that they signed up voluntarily for a summer professional development institute that focused on this practice, and nearly all stated that they thought it was useful to expose students to multiple strategies while teaching math. Thus, research about the implementation of educational policy provides us with a theoretical framework that motivates and helps to interpret the results of the present study.

As elaborated below, relevant research, textbooks, and teaching methods courses have developed and elaborated pathways for teachers to implement instruction with multiple strategies in their classrooms, building a ‘bridge’ to aid teachers in translating policy recommendations into classroom practices (e.g. Carroll, 2000; Fennema et al., 1996; Van de Walle, 2007). Yet as we also note, these research and practice efforts have focused largely on translating this policy into practice in the *elementary* classroom. How can we conceive of secondary teachers' views about a policy reform that was broadly targeted to teachers generally, but elaborated via research and practice recommendations largely for teachers in the lower grades?

Prior research suggests several strands of explanations which may bear on this question. First, early policy implementation research emphasized the importance of policy enactors' capacity and will to implement reforms, arguing that curricular reforms and new school policies are unlikely to be implemented consistently or in line with policymakers' visions when policy enactors lack needed training, or when they do not perceive the new reforms to be valuable (McLaughlin, 1987). In addition, some early research depicted policy enactors 'misinterpreting' policies intentionally due to disagreement with the policy or the desire to rationalize a preferred adaptation to the policy (Spillane, Reiser, & Reimer, 2002). However, more recent research has suggested that teachers' views and decisions around enacting a new policy reform, such as a curricular reform, should be considered from a cognitive perspective (Spillane, 1998, 2000). In this perspective, teachers are conceptualized as learners of the new reform, who must make sense of what the reform entails. In this model, as teachers gather information about the reform, they integrate it with their existing knowledge, beliefs, and understandings, in turn constructing their own interpretations, meanings, and ideas about the new policy (Spillane, 2000). Here teachers may mold their ideas about the new reform to fit their existing cognitive schemas and frameworks for understanding instruction, or, finding their existing schemas inadequate to integrate the new reform, reinterpret prior experiences or revise existing schemas in order to integrate the new reform ideas (Kolodner, 1997).

Building on the notion of teachers as learners of policy reforms, researchers have explored a range of mechanisms by which teachers learn about new instructional reforms. While these mechanisms are many and complex, one key finding that emerges from this research is the importance of non-system actors, or actors outside of the formal public policy system governing schools, such as textbook publishers, professional development providers, and university training programs; these have been found to be highly influential in shaping teachers' learning about and responses to instructional policy reforms (Coburn, 2005; Cohen & Hill, 2000). For example, in a study of teachers' responses to the implementation of new state curriculum standards in mathematics, Cohen and Hill (2001) found that the quality and content of the professional development that teachers received was associated with the extent to which they adopted instructional practices aligned with the reform. Similarly, in a study of teachers' responses to changes in state reading policies, Coburn (2005) found that policy reform messages that teachers received from non-system actors had a stronger influence on their instructional practice than did policy reform messages garnered from 'system actors,' such as standards documents, state and district frameworks, and government-produced policy documents. This may have been due in part to the higher degree of intensity of teachers' engagement with non-system actors, and to the fact that non-system actors were 'closer to the classroom,' providing teachers with textbooks and curriculum materials that they could utilize directly with students (Coburn, 2005).

In turn, noting the importance of non-system actors in shaping teachers' enactment of new instructional policies, Cohen and Hill (2001) have argued that in order for teachers to develop the knowledge and understandings they need to implement ambitious instructional reform policies in mathematics, teachers need to be provided with curriculum materials that are closely tied to the new reform recommendations, as well as professional development that is tied to these materials, grounded in student learning, and focused on classroom implementation. Without these resources, teachers may develop views on the reforms that are fragmented and distinct from those which developers of the reform intended to promote (Cohen & Hill, 2001).

In addition to the focus on teachers as learners, other policy implementation research has highlighted the importance for policy enactment of the implementation setting and context. For example, some researchers have characterized policy reform enactment in local contexts as a mutually adaptive process in which enactors of a policy reform adapt the new policy to meet local needs ‘on the ground,’ even as local conditions are simultaneously modified to accommodate the new policy (McLaughlin, 1990). Spillane, Reiser, and Reimer (2002) note that particularly at the secondary level, teachers’ professional specializations in a specific subject matter area and grade level comprise one important ‘context’ that shapes teachers’ interpretations and enactment of policy reforms. In a study on the contexts of secondary school teaching, Grossman and Stodolsky (1995; see also Stodolsky & Grossman, 1995) suggested that secondary mathematics teachers participated in a shared ‘subject subculture’ that held important similarities across schools, including common beliefs specific to the secondary mathematics context about subject matter content and norms for instructional practices. They found that teachers’ views about the potential of instructional reforms were shaped by their perceptions of the secondary mathematics curriculum as highly sequential, and requiring teachers to maintain a rapid instructional pace in order to ‘cover’ a well-defined set of required content (Grossman & Stodolsky, 1995). Secondary mathematics teachers viewed and interpreted policy reforms through the lens of the specific demands of upper-level mathematics curriculum and students, which they saw as posing unique demands of pacing, sequencing, and uneven student preparation (Grossman & Stodolsky, 1995). In turn, the ‘subject subculture’ context of secondary mathematics teaching may mediate secondary math teachers' response to instructional policy reforms (Grossman & Stodolsky, 1995; Spillane, Reiser, & Reimer, 2002).

In sum, teaching with multiple strategies can be viewed as a policy reform. Theoretically, this paper is centrally concerned with teachers' responses to this reform, with the goal of better understanding ways that teachers interpreted and made sense of the policy.

Research on Instruction with Multiple Strategies

A growing body of research suggests that students derive learning benefits from the practice of solving mathematics problems in more than one way (e.g., Gentner & Namy, 1999; Rittle-Johnson & Star, 2007; Silver et al., 2005; Star & Rittle-Johnson, 2008). Providing instruction incorporating multiple strategies has been identified in prior research as one of the fundamental principles that guide effective mathematics instruction (Gill & Thompson, 1995; Lampert, 1986; Leinhardt, 1987; Resnick, Bill, & Lesgold, 1992; Stein, Engle, Smith, & Hughes, 2008; Woodward et al., 2012), and case studies of expert teachers call attention to the importance of engaging students in comparing multiple strategies (Ball, 1993; Fraivillig, 1999; Hufferd-Ackles, Fuson, & Sherin, 2004; Lampert, 1990). For example, Atanga (2012) presented a case study of a fifth-grade teacher using the *Investigations in Number, Data, and Space* curriculum (TERC, 2008), who as a result of her professional development training effectively attended to the curriculum's 'teacher notes' on student discussions of multiple strategies, which she then utilized to lead a productive classroom discussion. Similarly, Bostic and Jacobbe (2010) and Ridlon (2009) described short-term, researcher-facilitated interventions with fifth- and sixth-grade students, respectively, which emphasized problem-based learning and the discussion of multiple strategies; in both cases, students appeared to experience learning gains and increased enthusiasm when learning with these approaches.

Encouraging students to generate, analyze, and discuss multiple strategies for solving problems is recommended as one (although certainly not the only) important component of effective mathematics instruction in NCTM's (2000) *Principles and Standards for School Mathematics*. More recently in the US, the *Common Core State Standards for Mathematics* (National Governors Association Center for Best Practices, 2010) state that students should be given opportunities to compare, discuss, and critique multiple strategies. Indeed, U.S. researchers and policymakers have argued for the importance of encouraging students to solve and discuss multiple strategies for solving mathematics problems for over two decades (Silver et al., 2005), and this practice has been recommended in policy reports in many countries (e.g., Australian Education Ministers, 2006; Brophy, 2000; Kultusministerkonferenz, 2004; National Council of Teachers of Mathematics, 2000; Singapore Ministry of Education, 2006; Treffers, 1991; Woodward et al., 2012).

Although the practice of teaching with multiple strategies appears to be ubiquitous in the research and practice arenas, a closer inspection of the literature reveals that this practice has been explored almost exclusively in the elementary grades. At the elementary level, the literature has placed emphasis on the importance of students' invention of their own strategies (as compared with teachers' demonstration of strategies), and on teachers' opportunities to assess their students' thinking by examining their diverse strategies (e.g. Fennema et al., 1996; Nelson, 2001; Yackel & Cobb, 1996). The literature further emphasizes the importance of teachers carefully sequencing the presentation of multiple strategies in order to build toward a mathematical point (Stein, Engle, Smith, & Hughes, 2008). Projects such as Cognitively Guided Instruction (CGI) played an early and important role in drawing elementary educators' attention to the issue of multiple strategies (Fennema et al., 1996) Case studies of effective implementation of CGI and related programs describe, for example, teachers guiding students to discuss the relative merits of their multiple invented strategies, to compare the similarities and differences between strategies, and to explain and justify why divergent methods might generate the same answer (Carpenter et al., 1999). Overall, there appears to be a consensus among researchers that there is a developmental appropriateness to accepting multiple strategies from young children, in that children are encouraged to begin solving problems using the intuitive mathematical knowledge that they have developed prior to entering the classroom, and to progress to strategies that are increasingly complex and abstract for the same sorts of problems.

Building on this research, the practice of teaching with multiple strategies is frequently encouraged in methods courses for elementary teaching. For example, in a text widely used in elementary mathematics teaching methods courses in the US (Van de Walle, 2007), teachers are encouraged to design problem solving tasks with "multiple entry points" (p. 51). Children are described as progressing from intuitive methods, such as adding using counters, to more sophisticated methods as their understanding of problem-solving strategies progresses (Van de Walle, 2007). Elementary school curricula also frequently emphasize students' invention of multiple strategies. For example, in one of the relatively widely used reform-oriented elementary curriculum programs in the United States, the University of Chicago School Mathematics Project's [UCSMP] *Everyday Mathematics* curriculum (Bell, Bell, & Hartfield, 1993), the materials emphasize that children should be given opportunities to invent their own strategies and to discuss and justify these with their peers (Carroll, 2000).

In addition to describing models of effective instruction with multiple strategies, the research literature has furthermore identified ways that such an approach can go awry. One key concern is that attempts to teach with multiple strategies may comprise mere "serial sharing," where students share multiple strategies but the teacher does not draw mathematical connections between them (Ball, 2001; Stein, Engle, Smith, & Hughes, 2008). For example, Stein, Engle, Smith, and Hughes (2008) present a vignette of “Mr. Crane,” a fourth-grade mathematics teacher who allowed multiple students to present and explain their solutions publicly for the class, but failed to drive these various presentations toward mathematical goals. Such a structure denies students a core benefit of the presentation of multiple strategies: the chance to compare, contrast, and evaluate the relative merits of the diverse strategies. When these connections are not made, the benefit that students derive from the presentation of multiple strategies is unclear (Ball, 2001).

To help teachers use students’ multiple strategies more effectively, Stein et al. (2008) have detailed a pedagogical model that includes anticipating, monitoring, selecting, sequencing, and making connections among students’ responses. In so doing, Stein et al. (2008) argue that key barriers to teachers’ enactment of effective discussions around multiple strategies have included a belief that teachers should avoid telling students anything, and a lack of direction for teachers in managing the pedagogical challenges associated with orchestrating classroom discussions that build toward important mathematical ideas. For example, in the domain of algebra, Smith (2011) examines evidence from the TIMSS video study and finds that in a U.S. eighth grade lesson on solving systems of equations -- a topic typically presented in multiple ways in U.S. math textbooks -- the teacher offered a highly procedural presentation, which did not afford students opportunities to make connections between multiple strategies and to examine algebraic relationships.

Multiple strategies in middle and secondary school

While there has been substantial research regarding teaching with multiple strategies in the elementary grades, there has been significantly less research about this practice at the middle and high school levels. Yet given the broad differences in the mathematical content, student characteristics, teacher demographics, and pedagogical approaches typically employed in mathematics courses at the middle and secondary level compared with at the elementary level, it seems reasonable to speculate that the perception of multiple strategies might also differ from the elementary to the secondary level. Indeed, a consideration of how teaching with multiple strategies might differ across grade levels raises several interesting questions. For example, what expectations for multiple strategy instruction might teachers report in the middle and high school grades? Would we expect such reports to mirror those found in elementary school, with, for example, high school teachers reporting that they expect their algebra students to generate and/or be exposed to more than one way to solve linear equations? Would we expect teachers to report expecting upper-level students to invent, share, and then compare multiple strategies for (for example) solving quadratic equations? How would we expect secondary teachers' reported goals for teaching with multiple strategies to be similar or different than those of elementary teachers?

These questions are largely unexplored in the literature (as we discuss more below); however, it is likely that speculative answers fall into two categories. On the one hand, one could argue in favor of teaching with multiple strategies in both elementary and secondary grades, essentially identifying a great deal of similarity in the goals for teaching with multiple strategies and the instructional practices for doing so. Our hunch is that this view—support for the importance of teaching with multiple strategies at all grade levels in mathematics—is widespread in the field. On the other hand, one could plausibly make a counter-argument: Despite solid evidence and broad consensus in favor of a focus on multiple strategies in the elementary grades, perhaps this approach is not particularly useful in the secondary grades. For example, given some teachers' perception that there is more content that needs to be "covered" in the secondary grades, perhaps there is not time to teach more than one strategy for a given problem. In addition, given the length and complexity of many of the strategies taught in secondary school, some teachers may feel that learning a single strategy is sufficiently challenging for their students—much less learning more than one strategy. Note that our aim here is not to argue against the use of multiple strategies, but rather to point out that the relevance of and utility of teaching with multiple strategies is largely unexplored outside of the elementary grades and thus a question worth investigating. Furthermore, despite widespread endorsement of incorporating multiple strategies in mathematics instruction, this practice is infrequently utilized in U.S. secondary classrooms (Silver et al., 2005).

We were able to identify only two studies that examined the views of middle and secondary school teachers on teaching with multiple strategies. In the first, Silver and colleagues (Silver et al., 2005) described the results of a professional development series conducted with twelve middle school mathematics teachers implementing the *Connected Mathematics* (Lappan, Fey, Fitzgerald, Friel, & Phillips, 2009) curriculum. Silver and colleagues found that almost all teachers in their sample expressed general support for the idea of teaching with multiple strategies at the beginning of their study, but that these teachers also expressed concerns and doubts about this approach both early in the study and as the study progressed. Among the key concerns that the teachers expressed were limited instructional time for presenting multiple strategies, the risk that lower-ability students would become confused, and the risk that displaying incorrect solution strategies could worsen students' misconceptions.

In the second study, Leikin and Levav-Waynberg (2007) interviewed twelve Israeli secondary school mathematics teachers with the goal of identifying which aspects of teacher knowledge were associated with (what the authors referred to as) multiple-solution connecting tasks (e.g., tasks that may be attributed to different topics or concepts within the mathematics curriculum, and therefore may be solved in multiple different ways). This study aimed to shed light on reasons for the disconnect between the theoretical evidence for the importance of teaching with multiple strategies, and teachers' limited use of this practice in their instruction. Leikin and Levav-Waynberg assessed teachers' subject matter knowledge by asking them to solve several problems in as many ways as they could, and to give an example of a mathematics problem that could be solved in different ways. They found that teachers' subject matter knowledge was "curriculum oriented," in that teachers generally described solution strategies that were taught in the Israeli school curriculum easily, but had greater difficulty coming up with additional solution strategies that were not prominent in the curriculum. In addition, when asked to produce an example of a problem that could be solved in multiple ways, teachers in their study frequently drew examples from topics for which students were required to learn multiple strategies in the Israeli school curriculum (such as solving systems of equations and or quadratic equations), or from their memories of student-generated solutions. Leikin and Levav-Waynberg also asked the Israeli teachers what they perceived the potential benefits of utilizing connecting tasks in teaching to be. Among the main benefits that teachers mentioned were that this practice could improve students' problem-solving success, provide affective advantages (e.g. helping students see the beauty in math, increasing their motivation and interest in math), and develop students' reasoning skills (e.g. developing logical reasoning, developing understanding).

Although these two studies are quite informative, we feel that exploration of this practice—teaching with multiple strategies in secondary school—merits further attention. The current study expands existing research on multiple strategies in middle and high school in several ways. First, the current study expands on the work of Leikin and Levav-Waynberg (2007) in that the primary focus of Leikin and Levav-Waynberg's study was on identifying those qualities of teachers' knowledge that supported their use of multiple strategies—what teachers knew about mathematics that did or did not allow them to teach with multiple strategies. The current study takes teachers' views on the practice of teaching with multiple strategies as its primary concern, seeking to understand how teachers themselves conceived of instruction with multiple strategies, and what goals for students' learning teachers described for multiple strategies instruction. The current study also extends the work of Silver and colleagues (Silver et al., 2005) in that the current study explores the views of a broader sample of both middle and high school teachers who used a variety of curricula, with 87 of the 92 teachers in the present study reporting use of traditional curricula that were not reform-oriented; the study from Silver and colleagues, by contrast, focused exclusively on the views of a group of middle school teachers who were already experienced and enthusiastic users of a reform-oriented curriculum that was by design conducive to the idea of teaching with multiple solution strategies.

In summary, the common and widely-endorsed practice of exposing students to multiple solution strategies for mathematics problems has not been given significant attention in middle and high school, and thus merits further exploration. In this paper, we begin our examination of this practice, in the context of a professional development institute whose goal was to encourage teachers to think more deeply and carefully about the use of multiple solution strategies in their instruction. We are particularly interested in teachers' initial views about the use of and value of multiple solution strategies, before participating in the professional development.

Method

In this section, we describe the professional development institute that provided the context for this study, data collection, and analysis.

Setting

Data for the current study were collected during three professional development workshops, held in July 2009 [Year 1], July 2010 [Year 2], and July 2011 [Year 3]. The content of all three workshops was the same; the goal of each workshop was to prepare participating teachers to implement a set of researcher-developed curriculum materials in their Algebra I courses. These curriculum materials were developed as part of an NSF-funded research project whose goal was to 'infuse' multiple strategies into Algebra I courses. (Elsewhere we report on the workshop more generally, as well as teachers' implementation of these curricula [Newton & Star, in press; Rittle-Johnson & Star, 2011]; here our interest focuses exclusively on teachers' views about multiple strategies prior to beginning the workshops.)

Participants

The workshop participants were 92 experienced Algebra I teachers from 70 schools located throughout the state of Massachusetts. Thirteen teachers participated in the first workshop in Year 1; 54 teachers participated in the second workshop in Year 2; and 25 participated in the third workshop in Year 3. Ninety of the teachers taught in public (87) or public charter (3) schools; all of these teachers had secondary certification in mathematics in Massachusetts, except for one teacher who was in the process of obtaining certification. The other two teachers were from private schools; neither was certified but both were quite experienced. The mean number of years of teaching experience for the 92 teachers was 10 (range 2 to 25); the mean age of teachers was 42 (range 24 to 66). Thirty-two of the teachers had bachelors' degrees in mathematics, and 61 had masters' degrees in education. The schools represented included a mix of urban, suburban, and rural schools.

Forty-one teachers taught courses predominantly serving high school students (ninth and tenth graders), and 51 taught courses predominantly serving middle school students (eighth and in a few cases seventh graders). Based on our analyses, there did not appear to be salient differences in teachers' interview and survey responses based on school type; as a result, describing these associations is not a focus of the current study. Eighty-seven of the teachers reported using a variety of traditional Algebra I curricula, and five reported using reform-oriented curricula (such as *University of Chicago School Mathematics Project Algebra* [University of Chicago School Mathematics Project, 2004]).

Teachers were recruited through print and e-mail advertisements seeking experienced Algebra I teachers who wanted to explore new techniques for teaching algebra. Teachers were paid an honorarium for their participation in the project.

Data Sources

In order to assess teachers' views about teaching with multiple strategies prior to their participation in the professional development workshop, interview data were collected for participants from Year 1, and survey data were collected for participants from Years 2 and 3. For Year 1 participants, semi-structured interviews were conducted with each of the 13 teachers, prior to the beginning of the professional development activities on the first morning of the Year 1 summer workshop. Teachers were asked whether they thought it was useful to expose students to multiple strategies while teaching math, what they thought the advantages and disadvantages were to this approach, and whether and how they used multiple strategies in their current teaching. The interviews began with a set of questions posed generally, asking teachers about their views on “exposing students to multiple strategies for solving a math problem.” In these questions, we left open the issue of whether teachers or students generated the multiple strategies, so that teachers could respond with their own interpretations of who could or should generate the multiple strategies. Teachers were interviewed individually in a private room; all interviews were audio-recorded and later transcribed. The interviews were conducted by the authors, as well as by other trained research assistants on the project. The interview was semi-structured; we began with a series of pre-determined questions but asked follow-up questions as appropriate. Each interview took approximately 30 minutes to complete. A copy of the interview protocol is provided in the Appendix.

For participants in Years 2 and 3, a computerized survey with questions nearly identical to those asked in the Year 1 interviews was administered to each of the participants prior to the beginning of the professional development activities on the first morning of the Years 2 and 3 summer workshops. (We administered surveys to all participants in years 2 and 3 as a matter of scale; due to resource constraints, we were unable to interview all teachers one-on-one in Years 2 and 3 as we did in Year 1.) The survey was administered electronically in a computer lab at the site of the professional development workshop. Teachers were given 45 minutes to complete the survey; all teachers finished in the allotted time. Teachers typed their responses to interview questions in a text box (which did not have a character limit).

Analysis

The analysis was conducted collaboratively, by both authors and an additional research assistant, with multiple researchers coding the data and engaging in discussions to resolve disagreements (Lincoln & Guba, 1985; Brantlinger, Jimenez, Klingner, Pugach, & Richardson, 2005). After transcription of the interviews was complete, we compiled the interview and survey data and independently listened to the interviews and read the survey responses. As we analyzed the responses, each researcher identified statements that were relevant to the research questions, and independently developed a list of codes for participants' responses based on his or her reading of the responses. Meetings were held to discuss emergent themes in the data, and to discuss the similarities and differences among the initial categories that each researcher had developed. The analysis proceeded via an iterative process, in which we revisited the transcripts and original data in multiple rounds, constantly comparing the data with the developed categories for teachers' responses and searching for evidence suggesting the need for revisions (Clement, 2000). We resolved all disagreements through discussion until we reached consensus.

Results

Our analysis of the interviews sought to answer the following research questions: (RQ1) What do middle and high school Algebra I teachers describe as the advantages of instruction that includes a focus on multiple strategies?; and (RQ2) What disadvantages to this practice do these teachers describe?. Our goal is to present general themes that emerged in teachers' responses. We begin with the first question, regarding teachers' views about the advantages of teaching with multiple solution strategies in middle and secondary mathematics classrooms.

Before beginning, however, it is worth noting that (as one might predict) at the beginning of the study, the idea of teaching with multiple strategies was already familiar to all of the teachers in the study. Indeed, in the interviews and surveys conducted at the beginning of the professional development, all of the teachers reported that they used multiple strategies in their teaching, and many reported using multiple strategies frequently. All but two of the 92 teachers stated that they thought that teaching with multiple strategies was useful, and about a fifth of the teachers (18%) reported using multiple strategies every day.

Teachers generally described this practice as different from the way that they themselves had learned math: 71% of the teachers reported that in their own learning, they were not frequently exposed to multiple strategies for solving math problems. As one teacher commented, “I am a product of the 1960's and 1970's student. At that time there was one and only one way of solving a problem!” Some teachers expressed negative views about the single-strategy instruction they had experienced, recalling, for example, the “awful math teachers,”; others recalled thriving under this approach, stating, for example, “That’s why I liked algebra so much, you know, it was because it had the rules, and you know, you followed the rules, and you came up with the right answer.”

Note that these responses were not surprising given our sample, which consisted of teachers who were participating voluntarily in a professional development institute that focused on teaching with multiple strategies in Algebra I. Indeed, because of the voluntary nature of our sample, we expected that the teachers participating in the current study were likely well above average compared with other teachers in their endorsement and implementation of teaching with multiple strategies. However, teachers expressed quite diverse views of what the advantages were to this approach.

(RQ1) What do middle and high school Algebra I teachers describe as the advantages of instruction that includes a focus on multiple strategies?

When teachers were asked what they saw as the advantages of exposing students to multiple strategies for solving a math problem, their responses fell into five general categories, with teachers expressing (1) an *individual student differences* orientation, indicating that multiple strategies helped students with different learning styles to find a method that worked best for their individual needs; (2) a *mathematical understanding* orientation, noting that multiple strategies developed students' mathematical thinking and problem solving skills or their understanding of the nature of mathematics as a domain permitting diverse strategies; (3) a *success* orientation, noting that exposing students to multiple strategies helped each student find one way to solve problems correctly; (4) an *affective*/*motivational* orientation, indicating that teaching with multiple strategies aided in areas such as student involvement and confidence; and/or (5) an *efficiency* orientation, expressing that exposing students to multiple strategies helped them learn to solve problems more quickly or efficiently. Some teachers' responses fell into multiple categories. We describe each of these types of responses below.

Individual student differences. The most-cited advantage of exposing students to multiple strategies was that this approach could accommodate what teachers perceived to be *individual student differences*. Seventy-one teachers (77%) gave responses relating to this category. These teachers often emphasized what they described as individual differences among students as necessitating instruction with multiple strategies, in order to ensure that at least one strategy was presented that "worked" for each individual student. Teachers very frequently made assertions such as "everyone learns differently" and "everyone thinks differently," and argued that teaching with multiple strategies increased their likelihood of presenting a strategy that matched each student's individual learning style and needs. In turn, teachers often suggested that their goal was to help each student find a single method that he or she could use to solve problems successfully.

In describing the individual differences that they discerned among their students, many middle and high school teachers in the current study appeared to rely on a "learning styles" framework that categorized students as, for example, visual, auditory, and kinesthetic learners, with 25% of teachers referencing such a framework. For example, Justin commented that "Students learn in a variety of ways. According to some this can be thought of as multiple intelligences, e.g., auditory, visual, kinesthetic. Since students learn and access information in a variety of ways, one will reach more students by offering multiple strategies of accessing information." Teachers frequently indicated that teaching with multiple strategies was useful because by providing students with multiple strategies, they were more likely to present a strategy that addressed each student's individual learning style.

Another source of individual differences among students that teachers described was their academic background and preparation for the Algebra I course. Many teachers noted that their student populations included students who were weak in prerequisite skills and underprepared for an Algebra I course, and commented that teaching with multiple strategies was necessary to help such students understand algebra concepts and solve algebra problems. In comments typical of several teachers, Jody stated that exposing students to multiple strategies was useful because "There may be something that triggers something in that student, that's going to allow them to enter that problem, whereas they may not be ready for something that you're showing them . . . but there may be another way around it." She further stated that "[I]t might not be the most effective way or the most strategic way, but it gives them an entry point." Furthermore, to increase comprehension among less-prepared students, teachers often stated that they would incorporate graphical and other visual tools, stories, and physical activities to model problems.

Mathematical understanding. About half of the teachers (52%) described deepening students' mathematical understanding, such as by developing students' logical thinking and problem solving skills, as an advantage of instruction with multiple strategies. These responses fell into four sub-categories: (1) *developing students' understanding, logical thinking, and/or* *problem solving skills*; (40%); (2) changing students' beliefs about math by helping them to *understand that math is not a rigid set of rules* (12%); (3) helping students recognize that *some solution strategies are better than others for different problems* (10%); and (4), helping students *make connections among mathematical concepts and topics* (8%).

Increasing the likelihood of success. Slightly less than half of teachers (43% of the total) indicated that they believed teaching with multiple strategies would increase students' success in solving problems or generating correct answers. These responses often overlapped with statements about individual differences among students; a common comment was that teaching with multiple strategies helped students with different learning styles or modes of thinking (particularly low-achieving students) to find "their method" or "the way that works for them."

Affective/motivational and efficiency orientations. Several teachers mentioned two additional categories of advantages, indicating that teaching with multiple strategies could improve students' *affect/motivation* (15%), such as by getting students more involved, building confidence, and reducing boredom or frustration; or help students *learn to solve problems more efficiently* (8%).

(RQ2) What disadvantages to this practice do these teachers describe?

Regarding the disadvantages of exposing students to multiple strategies for solving a math problem, teachers' responses fell into nine general categories: (1) *risk of confusion*; (2) *time constraints*; (3) *affective and motivational issues*, which were closely linked to (4) *students' resistance to learning multiple strategies* and (5) *students' beliefs about math*; (6) *teacher knowledge constraints;* (7) *physical resources constraints*; (8) *added difficulty to the teacher's job*; and (9) *lack of teacher knowledge of multiple strategies*. Note that we include in this category teachers' responses to the survey and interview questions that asked about disadvantages directly, as well as their responses to the survey and interview questions that asked more generally about their concerns about using this approach in their teaching. Some teachers' responses fell into multiple categories. We describe each of these types of responses below.

Risk of confusion. By far the most commonly cited teacher disadvantage to multiple strategies was the risk of student confusion, noted by 87% of teachers. Teachers often indicated that when students were confused, they would urge students to adopt the strategy that the students felt most at ease with. For example, Tara commented, “It can confuse some kids because then they can’t decide which method they want to choose, so, it can be. But I often stress to the kids, you need to see which strategy works for you.”

In addition to not knowing which strategy to choose, several teachers (15%) also stated that their students mixed up different strategies if they were exposed to multiple strategies. As one teacher stated, “In a whole-class instruction setting, teaching multiple strategies can confuse and/or overwhelm the struggling student. Even able students can incorrectly ‘mix and match’ methods early on in a wildly confusing way.” In another typical comment, Barbara commented, “I have seen students mix up pieces of different strategies after being exposed to different approaches. Then we must re-teach or correct their assumptions and have them focus on one method that makes sense to them.” About a third of the teachers (36%) cited a specific risk of confusion for low-achieving students.

Time constraints. After student confusion, the next most commonly cited disadvantage was a lack of time; nearly half of teachers (45%) described either the class time or the extra preparation time needed as prohibitive. Many teachers referenced the need to “cover” extensive material for standardized tests; for example, one teacher stated, “Sometimes we need to barrel through the material in order to 'get it all in' by the end of the year or before they need to apply certain concepts on standardized tests."

Students' resistance to learning multiple strategies. Nearly a third of teachers (30%) stated that their students resisted learning multiple strategies, and preferred to be told how to perform a single method that would lead to the correct answer. These teachers generally cited their students' preference for generating an answer rapidly and correctly, and what they described as their students' aversion to thinking in greater depth about a problem. Nina, for example, commented, "I think some kids are like, 'Okay just tell me how to do it.' Or 'Tell me *a* way to do it, and just, I'll do it that way, even if it makes no sense to me, you know, at all.'"

In comments closely linked to this disadvantage, six teachers (7%) stated that *students' beliefs about mathematics*, specifically their belief that there should be one "right way" to solve a math problem and their lack of understanding of why they should know more than one way to solve a math problem, could prove to be roadblocks to their learning of multiple strategies.

Affective and motivational issues. Also closely linked to student resistance and beliefs, affective and motivational issues were cited by several teachers (16%). Approximately 12% stated that a disadvantage was that high-achieving students might become bored. Derek, for example, stated that a primary disadvantage of exposing students to multiple strategies was "boring the students who understand the first method." Several teachers also cited students' motivation, stating that struggling students lose confidence or "give up" when they cannot understand multiple approaches (8%); feel "stressed" or pressured by having to learn more than one strategy (3%); and derive "comfort" from learning only one strategy (5%). In a comment typical of these teachers' responses, Charlotte stated that for students weak in prerequisite skills, "when given 1 choice and 1 choice only....do it this way....they find comfort."

Additional disadvantages. Three additional disadvantages were raised, but each was cited by only a handful of teachers. Two teachers cited *added difficulty to the teacher's job*, including that it was harder to grade students' assignments if they had used different strategies; and two teachers cited *resource constraints* such as a lack of materials needed to teach with multiple strategies (e.g. manipulatives, calculators, computers). Three teachers commented that *teacher knowledge constraints* could be a problem, in that their own knowledge of multiple approaches was limited. In a comment typical of these teachers, Lisa stated, "For some topics I am very familiar with other approaches but for other topics I only know one way to solve a problem." Lastly, five teachers reported that they saw no disadvantages to this approach.

Discussion

We begin by reviewing summary points for each of our research questions. We then discuss the implications of this research for research on mathematics teaching and learning.

Our first research question concerned secondary teachers' views on the advantages of exposing students to multiple strategies. In our interviews and surveys, many teachers stated that the primary advantage of teaching with multiple strategies was that it helped them tailor their instruction to what they perceived to be substantial individual differences among students in their learning styles and modes of thinking. These teachers expressed the view that exposing students with disparate learning styles to multiple strategies for solving mathematics problems increased their chances of 'reaching' their students and helped ensure that each student found a method that worked for them.

This finding differs somewhat from that reported in Leikin and Levav-Waynberg (2007), who found that in their sample of Israeli teachers, the most common advantage cited for multiple strategies was that it could foster students' success in problem-solving. While improving students' success and skills in solving problems were prominent in the responses of many teachers in the current study, the focus on catering to students' individual learning needs and styles was notably more salient.

Beyond a general agreement on the value of providing "multiple entry points" for students with different learning styles, teachers participating in the Silver et al. (2005) study held somewhat different views on why mathematics instruction should focus on multiple strategies than did teachers in the current study. For example, while teachers in the Silver et al. study highlighted "access to students' thinking" as a benefit of teaching with multiple strategies, noting that asking students to present multiple strategies offered teachers a critically important window into any misconceptions that their students may have held, none of the teachers in the current study highlighted this advantage. This may be attributed to differences in teachers' background and experiences in the two studies, particularly teachers' familiarity with implementing reform-oriented curricula. Recall that the Silver et al. teachers were experienced and enthusiastic users of a reform-oriented curriculum, while the majority of teachers in the present study taught using a variety of traditional curricula. (We return to this issue below.)

In our second research question, we explored teachers' views on the disadvantages to teaching with multiple strategies. In interviews and surveys conducted prior to the professional development, almost all teachers expressed general endorsement of the idea of having students consider multiple solution strategies for problems, as well as confidence in their ability to teach with multiple strategies. However, teachers also described a number of disadvantages to this practice. The primary disadvantage cited by teachers in our study was that students, particularly low-achieving students, might become confused when seeing multiple approaches. To a lesser extent, the teachers mentioned motivational issues (such as higher-ability students losing interest in a problem), and teacher constraints, such as a lack of time. Our findings are similar to those of Silver and colleagues (Silver et al., 2005), who found that teachers expressed concerns about the idea that lower-ability students could become confused by multiple methods and about a perceived lack of time for multiple approaches.

Implications

One motivation for the present study was to explore the degree that middle and secondary mathematics teachers' views about teaching with multiple strategies differed from those expressed or attributed to elementary teachers in the literature. Our results suggest that there may be substantial differences in teachers' views between middle/high school teachers and those commonly attributed to elementary teachers. First, middle and secondary teachers did not express views about multiple strategies that (based on the extant literature) are widely present and held at the elementary level. For example, the middle and secondary teachers in the current study generally did not express the view that the generation of multiple strategies is useful (perhaps even vital) and developmentally appropriate as students progress from intuitive to more formal strategies. Second and conversely, the middle and secondary teachers who participated in the current study expressed some views about the value of teaching with multiple strategies that have not been previously reported in the elementary-focused literature. For example, many of the middle and secondary teachers in the current study indicated that a primary goal for introducing multiple strategies was to provide multiple entry points to a given problem for students with different learning styles. Based on the existing literature, this rationale does not play a prominent role among elementary teachers who teach with multiple strategies.

How can we interpret this disconnect between what elementary teachers say about teaching with multiple strategies and what secondary teachers say? We hypothesize that several factors may contribute to these differences. First, it seems possible that structural differences in teachers’ time spent with students at the elementary and secondary levels might play a role in teaching with multiple strategies. For example, elementary school teachers often spend most or all of the school day with the same students, and thus they may feel that they know and understand their students quite well and can relatively easily navigate the time demands of including multiple strategies in instruction. By contrast, middle and high school teachers may spend only one period per day with each group of students (for mathematics class), and thus may feel that the time with their students that might be required to incorporate multiple strategies is more constrained. Furthermore, while pacing is a concern for mathematics teachers at all levels, secondary teachers’ particular concern with the need to ‘cover’ substantial content at a rapid pace (Grossman & Stodolsky, 1995) may contribute to their hesitation to dedicate time to multiple strategies.

More broadly, if one is an advocate of reform-oriented teaching approaches, one might argue that secondary teachers have a limited and incomplete view of the practice of teaching mathematics with multiple strategies. In this view, the conversation within the field and the teaching practices being enacted at the elementary level could be considered much further along in terms of adoption and implementation of reform practices, while the secondary grades remain more traditional in terms of practices and curricula. By this interpretation one might hope that perhaps in time, teaching and learning mathematics in secondary school will more closely resemble what happens in elementary schools, and thus the differences between elementary and secondary teachers' views about multiple strategies would diminish over time. In other words, perhaps teachers' divergent views may be a temporary artifact of where the mathematics education research and practice are in the development of reform-oriented practices at all levels.

In support of this idea that elementary and secondary teachers are generally at different places in their development of reform-oriented teaching practices, it is interesting to note that the primary advantage secondary teachers cite for teaching with multiple strategies can be considered a somewhat more teacher-centered version of what elementary teachers say. More specifically, many secondary teachers in the present study relied heavily on a learning styles framework to justify their use of multiple strategies—that a teacher should show students multiple ways of solving problems so that eventually one way will 'stick' and make sense to each individual student. As noted above, elementary teachers' vision for the use of multiple strategies may be seen as more student-centered, with a stronger focus on helping students to justify, compare, and investigate different strategies. (It should also be noted that many scholars feel that there is limited evidence in support of the idea of learning styles, particularly the argument that a student is more likely to be able to understand instruction that matches his/her learning style [Pashler, McDaniel, Rohrer, & Bjork, 2008]).

In line with this view, perhaps as Cohen and Hill (2001)'s argument suggests, middle and high school teachers have not yet received the same level of aligned curriculum support and professional learning scaffolding as elementary teachers have received to implement the curricular policy reform of teaching with multiple strategies. Returning to the conceptualization of teachers as 'learners' of a policy reform (Spillane, 1998, 2000), perhaps in contrast to elementary teachers, teachers at the secondary level have not yet been provided with opportunities to learn about the policy reform recommendation to teach with multiple strategies that are 'close' to instruction, such as examples of curriculum materials, textbook resources, and activities that could bring the reform recommendations to life in the classroom with students. Perhaps without this 'learning-oriented' guidance about how to 'bridge' multiple strategies policy recommendations to classroom practice at their grade level, secondary teachers are developing views about instruction with multiple strategies that rely more heavily on their *contextual* perceptions of the specific needs and demands of students and mathematical content at the secondary level (Grossman & Stodolsky, 2005). For example, their views about the policy may be linked to contextual concerns such as students' uneven preparation and the need to 'cover' material rapidly to prepare students for subsequent courses. In the 'learning-oriented' conception of policy implementation, as secondary teachers' opportunities to learn about multiple strategies instruction via aligned curriculum support that is 'close to the classroom' develop, perhaps middle and high school teachers' views and practices associated with multiple strategies may evolve to more closely match those of elementary teachers.

Alternatively, perhaps the disconnect between what elementary and secondary teachers say about teaching with multiple strategies should not be dismissed as a developmental artifact in the adoption of reform-oriented practices. Rather, perhaps there are important differences in the mathematics and in students at the elementary versus at the secondary school level that indicate that certain practices are more appropriate for one group than the other. In other words, perhaps teaching with multiple strategies is more important in the elementary grades than in middle and high school. According to this view, secondary students may be generally more comfortable with abstraction and thus may not need intuitive and invented strategies. Similarly, the strategies that secondary students learn are generally more complex (e.g., have more intermediate steps); in addition, secondary students come with more advanced and complex prior knowledge. As a result, one might argue that the role of strategy invention (which plays a critical role in the use of multiple strategies in the elementary grades) may be unimportant or even detrimental at the secondary level. For example, many secondary teachers would likely not expect their students to invent new strategies for solving complex problems (e.g., inventing a new method for solving a quadratic equation) in a reasonable amount of time, and even if their students could do so, one might question whether students would get the same value from this exercise as elementary students do from invention. More generally, it seems reasonable to ask whether some widely-used and supported teaching practices in the elementary grades are not as appropriate in the secondary grades, and vice versa.

A complementary argument might proceed as follows: Perhaps rather than posing that some instructional practices that are useful for elementary mathematics teaching have limited applicability for secondary teaching, instead it might be the case that a teaching practice is important for both elementary and secondary teachers but for different reasons. For example, there may be important reasons for teaching multiple strategies in middle and high school that are different from rationales given by elementary school teachers. In the elementary grades, it seems clear that teaching with multiple strategies has the goal of building on students' intuitive knowledge in a developmentally appropriate way; however, in secondary school it may be the case that teaching with multiple strategies has the goal of helping students to develop a more connected understanding of mathematics and to develop fluency in mathematics by becoming more flexible and adaptive problem solvers.

In either case, our research indicates that secondary teachers appear to articulate different goals for teaching with multiple strategies than those found in the literature on teaching with multiple strategies at the elementary level. The issue merits further exploration, to think carefully about whether teaching with multiple strategies in some form is useful for secondary students and, if so, what goals this practice might be seeking to address.

Limitations

A primary limitation of this study is that we did not observe teachers' actual classroom practice, but looked only at their descriptions of their views about teaching with multiple strategies. In future studies, it would be useful to observe teachers' practice in order to see how they implement a multiple strategies approach with students. Such observations could also provide an additional lens for understanding teachers' views on multiple strategies, beyond what can be captured via interviews and surveys. In addition, a limitation of the current study was that it did not provide teachers with prompts and opportunities to discuss how they would teach students to engage with non-routine problems, such as problems for which there is no common solution algorithm; posing these problems could have elicited different responses from teachers. This would be a valuable avenue for future research, in order to gauge middle and secondary teachers' use and ideas about multiple strategies in more diverse contexts.

Conclusions and Future Directions

Providing instruction with multiple strategies introduces a new challenge to teaching. At the middle and secondary school level, where limited research has been done on what teaching with multiple strategies does or should look like, the challenge is even more pronounced. Stein et al. (2008) identified teachers' belief that they should avoid telling students anything and lack of guideposts for orchestrating classroom discussions as barriers to teachers' effective instruction with multiple strategies. Here we find some suggestive evidence for the possibility that some middle and high school teachers' views about the goals of teaching with multiple strategies may pose an additional barrier. For example, one concern is that if teachers focus too closely on the goal of finding a strategy that 'works' for every student, their discussions may fall short of the National Research Council's (2001) goal that students should be able to utilize multiple approaches and adaptively select strategies appropriate to diverse types of problems. As Cohen and Hill (2001)'s research suggests, aligned curriculum and professional learning opportunities for secondary teachers surrounding instruction with multiple strategies may be needed to build teachers' capacity to enact these practices as the reform developers envisioned.

Middle and high school teachers in the current study described generally positive views of teaching with multiple strategies, and the desire to incorporate multiple approaches in their teaching. All indeed stated that they were attempting to do so. However, some middle and high school teachers' descriptions of their goals and views about exposing students to multiple solution strategies appear to diverge in important ways from those widely cited and recommended in the elementary literature. We believe the early, exploratory findings reported in the current study suggest the need for further investigation in this area, in order to understand middle and high school teachers' use of multiple solution strategies, and to develop strategies to support middle and high school teachers' engaging students in substantial thinking about multiple approaches in their classrooms.

References

Ambrose, R., Clement, L., Philipp, R. A., & Chauvot, J. (2004). Assessing prospective elementary school teachers' beliefs about mathematics and mathematics learning: Rationale and development of a constructed-response-format beliefs survey. *School Science and Mathematics, 104*(2), 56-69. doi: 10.1111/j.1949-8594.2004.tb17983.x

Atanga, N. A. (2012). Understanding how an elementary teacher recognizes and uses curriculum features. In L. R Van Zoest, J.-J. Lo, & J. L. Kratky (Eds.), *Proceedings of the 34th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Kalamazoo, MI: Western Michigan University.

Australian Education Ministers. (2006). *Statements of learning for mathematics*. Carlton, South Victoria, Australia: Curriculum Corporations.

Ball, D. L. (1993). With an eye on the mathematical horizon: Dilemmas of teaching elementary school mathematics. *Elementary School Journal, 93*(4), 373-397.

Ball, D. L. (2001). Teaching, with respect to mathematics and students. In T. Wood, B. S. Nelson & J. Warfield (Eds.), *Beyond classical pedagogy: Teaching elementary school mathematics* (pp. 11-21). Mahwah, NJ: Erlbaum.

Bell, M., Bell, J., & Hartfield, R. (1993). *Everyday Mathematics*. Evanston, IL: Everyday Learning Corporation.

Bostic, J. D., & Jacobbe, T. (2010). Facilitating use of multiple strategies in a high poverty classroom through problem-based instruction. In P. Brosnan, D. B. Erchick, & L. Flevares (Eds.), *Proceedings of the 32nd annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Columbus, OH: The Ohio State University.

Brantlinger, E., Jimenez, R., Klingner, J., Pugach, M., & Richardson, V. (2005). Qualitative studies in special education. *Exceptional Children*, *71*, 195–207. doi: 10.1207/s15327035ex0502_1

Brophy, J. (2000). *Teaching*. Brussels, Belgium: International Academy of Education.

Cai, J., & Knuth, E. (Eds.). (2011). *Early algebraization: A global dialogue from multiple perspectives*. Heidelberg, Germany: Springer.

Carpenter, T. P., Fennema, E., Fuson, K., Hiebert, J., Human, P., Murray, H., . . . Wearne, D. (1999). Learning basic number concepts and skills as problem solving. In E. Fennema & T. A. Romberg (Eds.), *Mathematics classrooms that promote understanding* (pp. 45-61). Mahwah, NJ: Erlbaum.

Carpenter, T. P., Franke, M. L., Jacobs, V. R., Fennema, E., & Empson, S. B. (1998). A longitudinal study of invention and understanding in children's multidigit addition and subtraction. *Journal for Research in Mathematics Education, 29*(1), 3-20.

Carroll, W. M. (2000). Invented computational procedures of students in a standards-based curriculum. *Journal of Mathematical Behavior*, *18*(2), 111-121. doi: 10.1016/S0732-3123(99)00024-3.

Clark, C. M., & Peterson, P. L. (1986). Teachers' thought processes. In M. Wittrock (Ed.), *Handbook of research in teaching*. New York, NY: MacMillan.

Clement, J. (2000). Analysis of clinical interviews: Foundations and model viability. In A. E. Kelly & R. A. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 547-589). Mahwah, NJ: Erlbaum.

Coburn, C. E. (2005). The role of nonsystem actors in the relationship between policy and practice: The case of reading instruction in California. *Educational Evaluation and Policy Analysis*, *27*, 23-52. doi: 10.3102/01623737027001023

Cohen, D. K., & Hill, H. C. (2000). Instructional policy and classroom performance: The mathematics reform in California. *Teachers College Record*, *102*(2), 294-343. doi: 10.1111/0161-4681.00057

Cohen, D. K., & Hill, H. C. (2001). *Learning policy: When state education reform works*. New Haven, CT: Yale University Press.

Fang, Z. (1996). A review of research on teacher beliefs and practices. *Educational Research*, *38*(l), 47–65. doi: 10.1080/0013188960380104

Fennema, E., Carpenter, T. P., Franke, M. L., Levi, L., Jacobs, V. R., & Empson, S. B. (1996). A longitudinal study of learning to use children's thinking in mathematics instruction. *Journal for Research in Mathematics Education*, *27*, 403-434.

Fraivillig, J. L., Murphy, L., & Fuson, K. C. (1999). Advancing children's mathematical thinking in reform mathematics classrooms. *Journal for Research in Mathematics Education*, *30*(2), 148-170.

Gentner, D., & Namy, L. L. (1999). Comparison in the development of categories. *Cognitive Development*, *14*(4), 487-513. doi: 10.1016/S0885-2014(99)00016-7

Gill, A. J., & Thompson, A. (1995). Bridging second grade children's thinking and mathematical recording. *Journal of Mathematical Behavior*, *14*, 349-362. doi: 10.1016/0732-3123(95)90016-0

Grossman, P. L., & Stodolsky, S. S. (1995). Content as context: The role of school subjects in secondary school teaching. *Educational Researcher*, *24*, 5-23. doi: 10.3102/0013189X024008005

Hollingsworth, S. (1989). Prior beliefs and cognitive change in learning to teach. *American Educational Research Journal*, *26*, 160-189. doi: 10.3102/00028312026002160

Hufferd-Ackles, K., Fuson, K., & Sherin, M. G. (2004). Describing levels and components of a math-talk learning community. *Journal for Research in Mathematics Education*, *35*(2), 81-116.

Izsák, A., Çağlayan, G., & Olive, J. (2009). Meta-representation in an Algebra I classroom. *The Journal of the Learning Sciences*, *18*(4), 549–587. doi: 10.1080/10508400903191912

Kagan, D. M. (1992). Implications of research on teacher belief. *Educational Psychologist*, *27*, 65-90. doi: 10.1207/s15326985ep2701_6

Knuth, E. J., Alibabi, M. W., McNeil, N. M., Weinberg, A., & Stephens, A. (2011). Middle school students' understanding of core algebraic concepts: Equivalence and variable. In J. Cai & E. Knuth (Eds.), *Early algebraization: A global dialogue from multiple perspectives* (pp. 511-528). Heidelberg, Germany: Springer.

Kolodner, J. L. (1997). Educational implications of analogy: A view from case-based reasoning. *American Psychologist*, *52*(1), 57-66. doi: 10.1037/0003-066X.52.1.57

Kultusministerkonferenz [Standing Conference of the Ministers of Education and Cultural Affairs of the Länder in the Federal Republic—KMK]. (2004). *Bildungsstandards im fach mathematik für den primarbereich [educational standards in mathematics for primary schools]*. Luchterhand: München-Neuwied.

Lampert, M. (1986). Knowing, doing, and teaching multiplication. *Cognition and Instruction, 3*(4), 305-342. doi 10.1207/s1532690xci0304_1

Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. *American Educational Research Journal, 27*(1), 29-63. doi: 10.3102/00028312027001029

Lappan, G., Fey, J., Fitzgerald, W., Friel, S., & Phillips, E. (2009). *Connected mathematics*. Upper Saddle River, NJ: Pearson Education.

Leikin, R., & Levav-Waynberg, A. (2007). Exploring mathematics teacher knowledge to explain the gap between theory-based recommendations and school practice in the use of connecting tasks. *Educational Studies in Mathematics, 66*(3), 349-371. doi: 10.1007/s10649-006-9071-z

Leinhardt, G. (1987). Development of an expert explanation: An analysis of a sequence of subtraction lessons. *Cognition and Instruction, 4*, 225-282. doi: 10.1207/s1532690xci0404_2

Lincoln, Y. S., & Guba, E. G. (1985). *Naturalistic inquiry*. Beverly Hills, CA: Sage.

McLaughlin, M. W. (1987). Learning from experience: Lessons from policy implementation. *Educational Evaluation and Policy Analysis*, *9*(2), 171-178. doi: 10.3102/01623737009002171

McLaughlin, M. W. (1990). The Rand change agent study revisited: Macro perspectives and micro realities. *Educational Researcher*, *19*, 11-16. doi:10.3102/0013189X019009011

Moses, R., & Cobb, C. (2001). *Radical equations*. Boston, MA: Beacon Press.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010). *Common Core State Standards for Mathematics*. Washington, DC: Author.

National Research Council. (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.

Nelson, B. S. (2001). Constructing facilitative teaching. In T. Wood, B. S. Nelson, & J. Warfield (Eds.), *Beyond classical pedagogy: Teaching elementary school mathematics* (pp. 251-273). Mahwah, NJ: Erlbaum.

Newton, K. J., & Star, J. R. (in press). Exploring the nature and impact of model teaching. *Mathematics Teacher Educator*.

Pajares, M. F. (1992). Teachers' beliefs and educational research: Cleaning up a messy construct. *Review of Educational Research*, *62*(3), 307-332. doi: 10.3102/00346543062003307

Pashler, H., McDaniel, M., Rohrer, D., & Bjork, R. (2008). Learning styles: Concepts and evidence. *Psychological science in the public interest*, *9*(3), 106-116. doi: 10.1111/j.1539-6053.2009.01038.x

Peterson, P. L., Fennema, E., Carpenter, T. P., & Loef, M. (1989). Teachers' pedagogical content beliefs in mathematics. *Cognition and Instruction*, *6*(1), 1-40. doi: 10.1207/s1532690xci0601_1

Philipp, R. A. (2007). Mathematics teachers' beliefs and affect. In F. Lester (Ed.), S*econd handbook of research on mathematics teaching and learning*. Reston, VA: National Council of Teachers of Mathematics.

Philipp, R. A., Ambrose, R., Lamb, L. L. C., Sowder, J. T., Schappelle, B. P., Sowder, L., Thanheiser, E., & Chauvot, J. (2007). Effects of early field experiences on the mathematical content knowledge and beliefs of prospective elementary school teachers: An experimental study. *Journal for Research in Mathematics Education, 35*(5), 438–476.

Resnick, L. B., Bill, V., & Lesgold, S. (1992). Developing thinking abilities in arithmetic class. In M. S. A. Demetriou & A. Efkides (Eds.), *The modern theories of cognitive development go to school*. London, England: Routledge.

Richardson, V. (1990). Significant and worthwhile change in teaching practice. *Educational Researcher*, *19*(7), 10-18. doi: 10.3102/0013189X019007010

Richardson, V., Anders, P., Tidwell, D., & Lloyd, C. (1991). The relationship between teachers' beliefs and practices in reading comprehension instruction. *American Educational Research Journal*, *28*(3), 559-586. doi: 10.3102/00028312028003559

Ridlon, C. L. (2009). Learning mathematics via a problem-centered approach: A two-year study. *Mathematical Thinking and Learning*, *11*(4), 188-225. doi: 10.1080/10986060903225614

Rittle-Johnson, B., & Star, J. R. (2007). Does comparing solution methods facilitate conceptual and procedural knowledge? An experimental study on learning to solve equations. *Journal of Educational Psychology, 99*(3), 561-574. doi: 10.1037/0022-0663.99.3.561

Rittle-Johnson, B., & Star, J. R. (2011). The power of comparison in learning and instruction: Learning outcomes supported by different types of comparisons. In J. P. Mestre & B. H. Ross (Eds.), *Psychology of Learning and Motivation*: *Cognition in Education* (vol. 55) (pp. 199-222). Waltham, MA: Elsevier. doi: 10.1016/B978-0-12-387691-1.00007-7

Silver, E. A., Ghousseini, H., Gosen, D., Charalambous, C., & Strawhun, B. (2005). Moving from rhetoric to praxis: Issues faced by teachers in having students consider multiple solutions for problems in the mathematics classroom. *Journal of Mathematical Behavior, 24*(3-4), 287-301. doi: 10.1016/j.jmathb.2005.09.009

Singapore Ministry of Education. (2006). *Secondary mathematics syllabuses*.

Smith, M. (2011). A procedural focus and a relationship focus to algebra: How U.S. teachers and Japanese teachers treat systems of equations. In J. Cai & E. Knuth (Eds.), *Early algebraization: A global dialogue from multiple perspectives* (pp. 511-528). Heidelberg, Germany: Springer.

Spillane, J. P. (1998). A cognitive perspective on the role of the local educational agency in implementing instructional policy: Accounting for local variability. *Educational Administration Quarterly, 34*(1), 31-57. doi: 10.1177/0013161X98034001004

Spillane, J. P. (2000). Cognition and policy implementation: District policymakers and the reform of mathematics education. *Cognition and Instruction, 18*(2), 141-179. doi: 10.1207/S1532690XCI1802_01

Spillane, J. P., Reiser, B. J., & Reimer, T. (2002). Policy implementation and cognition: Reframing and refocusing implementation research. *Review of Educational Research, 72*, 387-431. doi: 10.3102/00346543072003387

Star, J. R., & Rittle-Johnson, B. (2008). Flexibility in problem solving: The case of equation solving. *Learning and Instruction, 18*, 565-579. doi: 10.1016/j.learninstruc.2007.09.018

Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning, 10*(4), 313-340. doi: 10.1080/10986060802229675

Stodolsky, S. S., & Grossman, P. L. (1995). The impact of subject matter on curricular activity: An analysis of five academic subjects. *American Educational Research Journal, 32*, 227-249. doi:10.3102/00028312032002227

Stylianou, D. A. (2010). Teachers' conceptions of representation in middle school mathematics. *Journal of Mathematics Teacher Education, 13*(4), 325-343. doi: 10.1007/s10857-010-9143-y

TERC. (2008). *Investigations in number, data, and space: Equal groups curriculum unit (Grade 3, Unit 5)* (2nd ed.). Glenview, IL: Pearson Scott Foresman.

Thompson, A. G. (1984). The relationship of teachers' conceptions of mathematics and mathematics teaching to instructional practice. *Educational Studies in Mathematics*, *15*(2), 105-127. doi: 10.1007/BF00305892

Thompson, A. G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning*. New York, NY: Macmillan.

Treffers, A. (1991). Didactical background of a mathematics program for primary education. In L. Streefland (Ed.), *Realistic mathematics education in primary school* (pp. 21-56). Utrecht, The Netherlands: Freudenthal Institute.

University of Chicago School Mathematics Project. (2004). *University of Chicago School Mathematics Project algebra.* Chicago, IL: McGraw-Hill.

Van de Walle, J. A. (2007). *Elementary and middle school mathematics: Teaching developmentally* (6th ed.). Boston, MA: Pearson.

Wilkins, J. L. M. (2008). The relationship among elementary teachers' content knowledge, attitudes, beliefs, and practices. *Journal of Mathematics Teacher Education, 11*(2), 139-164. doi: 10.1007/s10857-007-9068-2

Woodward, J., Beckmann, S., Driscoll, M., Franke, M. L., Herzig, P., Jitendra, A. K., . . . Ogbuehi, P. (2012). *Improving mathematical problem solving in grades 4 to 8: A practice guide (NCEE 2012-4055)*. Washington, DC: National Center for Education Evaluation and Regional Assistance, Institute of Education Sciences, U.S. Department of Education.

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education*, *27*(4), 458-477. doi: 10.2307/749877

Appendix

Interview protocol

1. As you know, in this project we are interested in whether students learn math when they are exposed to multiple strategies for solving a problem. Do you think it is useful to expose students to multiple strategies while teaching math?

2. What do you see as the advantages of exposing students to multiple strategies for solving a math problem?

3. What do you see as the disadvantages of exposing students to multiple strategies for solving a math problem?

4. In your own learning, were you frequently exposed to multiple strategies for solving math problems?

5. Which groups of students do you think might benefit more from this approach and why?

6. Which groups of students do you think might benefit less from this approach and why?

7. In your instruction, do you use multiple strategies to solve a problem?

   7a. *If yes, ask:* How often do you use it?

8. Can you give me an example from your teaching where you used multiple strategies?

9. What concerns do you have about regularly using comparison in your teaching?

To help us learn more about your current thinking about teaching multiple strategies, let's talk specifically about some math problems.

10. Let’s imagine you are in your classroom trying to explain students how to solve a linear equation such as $3(x + 2) = 21$. What strategy or strategies would you teach students to use for this problem?

10a. Are there other ways to solve this equation that you would teach to students?

10b. Can you tell me more about how you teach students these multiple approaches?

10c. Do you tell students that one strategy is better than another?

10d. Do you teach students that there are certain situations or problems where one strategy is better than another?

10e. Do you require students to know (such as on a test) multiple strategies for this problem?

11. Let's move to another problem. If you are trying to explain students how to simplify an expression such as $\sqrt{\frac{75}{3}}$, what strategy or strategies would you teach students to use?

11a. Are there additional ways to simplify this expression that you would teach to students?

11b. Can you tell me more about how you teach students these multiple approaches?

11c. Do you tell students that one strategy is better than another?

11d. Do you teach students that there are certain situations or problems where one strategy is better than another?

11e. Do you require students to know (such as on a test) multiple strategies for this problem?

12. And now a third problem. If you are teaching how to graph an equation, such as 6x + 3y = 12, what strategy or strategies would you teach students to use?

12a. Are there additional ways to graph this equation that you would teach to students?

12b. Can you tell me more about how you teach students these multiple approaches?

12c. Do you tell students that one strategy is better than another?

12d. Do you teach students that there are certain situations or problems where one strategy is better than another?

12e. Do you require students to know (such as on a test) multiple strategies for this problem?

13. Finally, if you are trying to explain to students how to solve a system of linear equations, such as $\begin{cases} 2x + y = 1 \\ x + y = 3 \end{cases}$, what strategy or strategies would you teach students to use?

13a. Are there extra ways to solve this system that you would teach to students?

13b. Can you tell me more about how you teach students these multiple approaches?

13c. Do you tell students that one strategy is better than another?

13d. Do you teach students that there are certain situations or problems where one strategy is better than another?

13e. Do you require students to know (such as on a test) multiple strategies for this problem?